B. The Demagnetization Curve and its Parameters

In hard magnetic materials the second quadrant of hysteresis is most important and the major curve there is called **demagnetization curve.** Demagnetization curves as well as all other curves in all quadrants of hysteresis can be drawn both the in J(H) or in the B(H) description, which follows from eq. (A.8). This is also the case in Fig. B1, which provides those basic parameters of the demagnetization curve, which are mainly used in technical literature about permanent magnets.
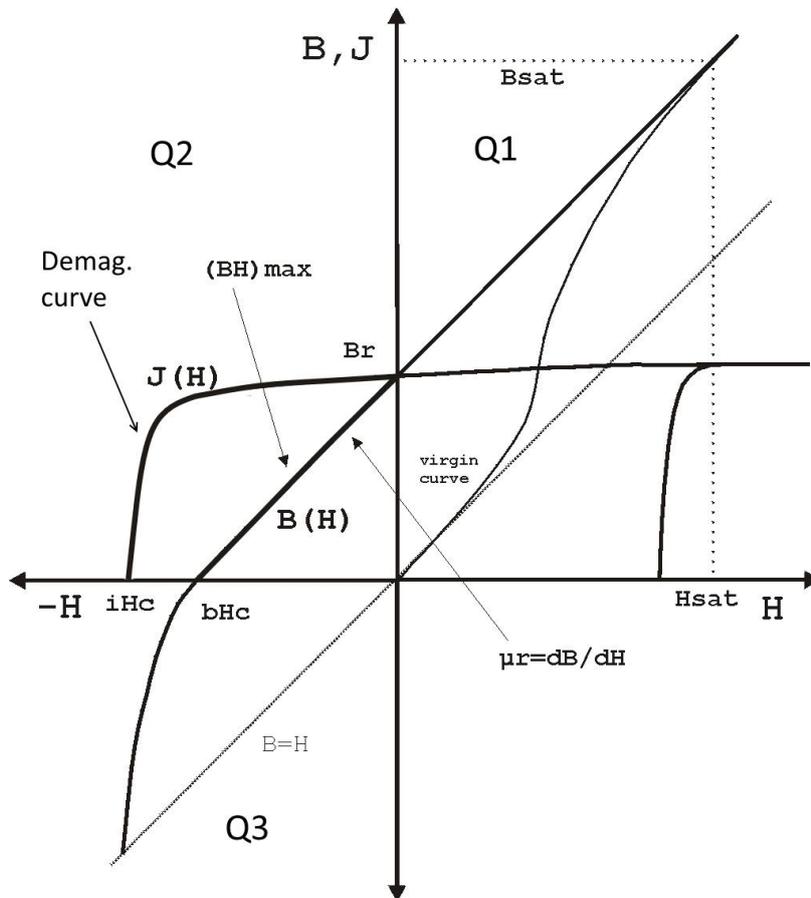

Fig.B1: Demagnetization curve as well as the first and parts of the third quadrant of magnetic hysteresis. The demagnetization curve, i.e. the main curve in the second quadrant, defines the parameters B_r, $_bH_c$, $_jH_c$, μ_r and $(BH)_{max}$.

The most important parameters of a demagnetization curve are named as:

B_r = **Remanence induction** [T]
$_jH_c$ = **Coercitivity of J** [A/m], $_bH_c$ = **Coercitivity of B** [A/m]
μ_r = **Recoil permeability** [no units]
$(BH)_{max}$ = **Maximum energy product** [kJ/m^3]

Now let's describe the behavior of a demagnetization curve in more detail. As we examine here only one spatial direction, a scalar description is used.

In modern magnetic materials we have a more or less linear behavior of J(H) and B(H) of the demagnetization curve up to a point where the curve bends down more sharply. If the magnets working points are located in this linear area, these locations can be moved by external H changes without leaving the original curve. The behavior of the magnet is then called to be reversible

In the M(H) picture the linearity of the demagnetization curve is described by introducing a constant recoil susceptibility χ_r This is now a differential susceptibility dM/dH, i.e. the steepness of the demagnetization in that region.

$$M(H) = M_r + \chi_r \cdot H \tag{B.1}$$

Here M_r is the remanent magnetization. Using eq. (A.2) we get for J(H)

$$J(H) = B_r + \mu_0 \cdot \chi_r \cdot H \tag{B.2}$$

From this we derive that the remanence induction in fig. B1 is related to the remanence magnetization simply by the factor μ_0:

$$B_r = \mu_0 \cdot M_r \tag{B.3}$$

In the B(H) description it follows from eq. (A.7) that:

$$B(H) = Br + \mu_0 \cdot \mu_r \cdot H \tag{B.4}$$

Here we have introduced the recoil permeability (or often called permanent permeability) by

$$\mu_r = 1 + \chi_r \tag{B.5}$$

The recoil permeability describes the steepness of the demagnetization curve in the B(H) depiction. It is also a differential parameter dB/dH here, in contradiction to the total permeability which was introduced in chapter A. The above formulas are not only valid for linear demagnetization curves but can also be used, when there is a deviation from linearity. In this case μ_r and χ_r are H dependent.

From the above equations it can be taken, that the remanence induction is nearly equal to μ_0 times the magnetization almost over the whole linear or quasilinear range of a demagnetization curve. This can be derived especially from eq. (B.1), as χ_r is close to zero (μ_r close to one) for most modern magnetic materials. When taking into account that the spatial distribution of magnetization determines the field of permanent magnets, see chapter E, the importance of remanence induction can easily be understood.

The coercivity of B i.e. $_bH_c$ describes that magnetic field, which makes the B distribution inside the magnet change its direction. It is smaller than $_iH_c$, which is the field necessary to demagnetize the polarization or magnetization to zero. The saturation field h_{sat}, which can be found in the first quadrant, is the field needed to magnetize the magnet into saturation.

The Maximum Energy Product, i.e. the point on the demagnetization curve were the product B*H has its maximum, is introduced mainly for purposes of comparison, since the energy of a magnetic field is given by:

$$E = \frac{1}{2} \int \vec{B} \cdot \vec{H} dV \qquad (B.6)$$

This means e.g. that the maximum field energy of an ideal magnetic circuit with a permanent magnet is at its maximum, when B*H on the demagnetization curve of the magnet is at its maximum. When the permanent magnet behaves linearly with a constant recoil permeability μ_r the maximum energy product can be expressed simply by:

$$(BH)_{max} = \frac{1}{4 \cdot \mu_r \cdot \mu_0} \cdot B_2{}^2 \qquad (B.7)$$

Up to now we have learned about different parameters of the demagnetization curve as well as some mathematical descriptions of its behavior. The origin for the importance of the demagnetization curve can be found in the fact, that all single magnets as well as most magnets in magnetic systems have their (B,H)-working points in the second quadrant of hysteresis. For the case of a single magnet this can be easily understood by the general difference between the **B**- and the **H**-fields. Fig. B2 shows the **B** as well as the **H** distribution of a single homogeneously magnetized sphere. Whereas the **B** field forms closed loops and is outside the magnet equal to $\mu_0\mathbf{H}$ ($\mu=1$ in air, comp. to eq. (A.11)), inside the sphere the **H** field is reversed in comparison to **B**. So the internal field **H** tends to demagnetize its own source and is located at the negative branch of hysteresis.
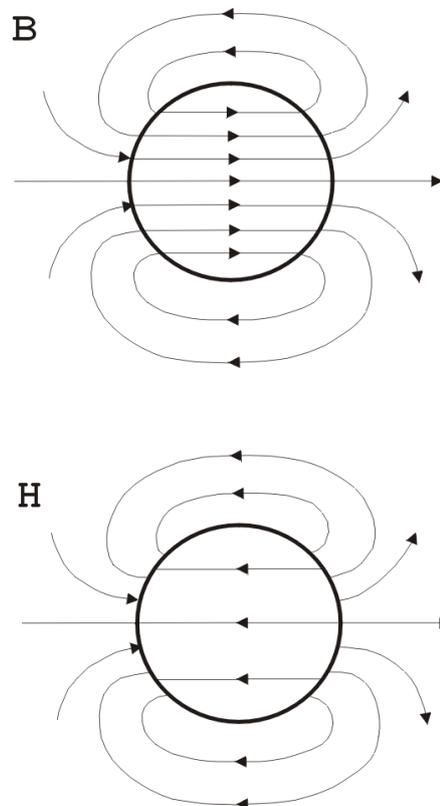

Fig.B2: Field distributions **B** and **H** around a homogeneously magnetized sphere.

For isolated magnets in case of ellipsoids the internal H field can be computed with the help of the **demagnetization factor** N:

$$\vec{H} = -N \cdot \vec{M} \qquad\qquad (B.8)$$

0 <= N <= 1, N ~>1: oblate ellipsoid, N~>0: prolate ellipsoid. Sphere: N=1/3.
For other magnets either an estimate of demagnetization factors can be given, if their geometry is close to an ellipsoid, or there is a distribution of different working points, i.e. different H fields inside body. Generally it can be stated that the length to diameter ratio (L/D) determines the strength of **H** (L parallel to direction of **M**):

$$L/D \gg 1: H \sim> 0, \; L/D \ll 1: \; H \text{ is large} \qquad\qquad (B.9)$$

In magnetic systems consisting of more entities than only one permanent magnetic field source, in most cases the remaining demagnetizing fields inside magnets are still negative, even if the external fields may act in the opposite direction. So also here the operating points are still located in the second quadrant of hysteresis, i.e. on the demagnetization curve.