A. Basic Parameters of Permanent Magnetism

Magnetic materials can be described as an assembly of magnetic moments, which are the source of magnetic fields. The magnetic moment is a vector quantity. Summing up all magnetic moments and dividing by the volume results in the so called magnetization vector. The magnetic moment can be considered analogous to the electric charge in electricity, the magnetization would then be equivalent to the electric charge density. So we have as basic carriers of magnetism

$\vec{m}$ magnetic Moment [Am^2]

$$\vec{M} = \frac{d\vec{m}}{dV} \quad \text{Magnetization [A/m]} \tag{A.1}$$

Instead of the magnetization vector literature and technicians often use the so called magnetic polarization **J**, which differs from **M** only by the permeability constant μ_0:

$$\vec{J} = \mu_0 \cdot \vec{M} \quad [T] \quad \text{with} \quad \mu_0 = 1.257 \cdot 10^{-6}\,\text{Vs/Am} \tag{A.2}$$

The resulting fields in magnetism can be divided into:

$\vec{B}$ magnetic Flux Density [T]

and

$\vec{H}$ magnetic Field Intensity [A/m]

The description of the fields and the resulting phenomena like forces, torques, energies etc. is done by using the **Maxwell equations**:

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{(Flux conservation)} \tag{A.3}$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \quad \text{(Amperes Law)} \tag{A.4}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faradays Law)} \tag{A.5}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{(Coulomb Law)} \tag{A.6}$$

j is the current density [A/m^2] and **E** denotes the electric field strength [V/m]. The electric flux density **D** has the unit [As/m^2]. ρ describes the electric charge density [As/m^3].

How enter magnetic moments or the magnetization vector these Maxwell equations ? This is the case by the so called **constitutive relation**, which supplies the relation between the fields **B**, **H** and the magnetization **M**:

$$\vec{B} = \mu_0 \cdot \vec{H} + \mu_0 \cdot \vec{M} \qquad \text{(A.7)}$$

Using J instead of M this becomes

$$\vec{B} = \mu_0 \cdot \vec{H} + \vec{J} \qquad \text{(A.8)}$$

The behavior of M, especially its dependence of H forms the division of magnetism into its basic phenomena. This H dependence is described by the so called susceptibility χ which is a unitless parameter:

$$\vec{M} = \chi\,(H) \cdot \vec{H} \qquad \text{(A.9)}$$

Defining the relative Permeability by

$$\mu = \chi + 1 \qquad \text{A.10)}$$

we get from eqs. (A.2), (A.8) and (A.9):

$$\vec{B} = \mu_0 \cdot \mu \cdot \vec{H} \qquad \text{(A.11)}$$

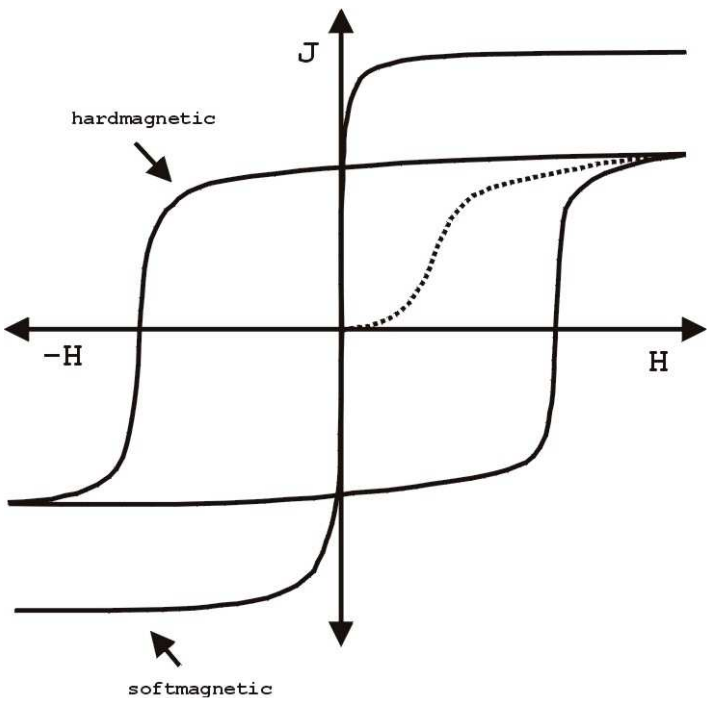

Fig.A1: Hard magnetic and soft magnetic materials and their J(H)-dependence. Doted curve: virgin magnetization curve.

We divide magnetism now into:

Diamagnetism: $\chi < 0$, $|\chi| << 1$ (A.12a)
Paramagnetism: $\chi > 0$, $|\chi| << 1$ (A.12b)

Ferromagnetism: $\chi > 0$, $|\chi| >= 1$ (A.12c)

Technically dia- and paramagnetism can be neglected in most cases, their background will be described in the area dealing with the atomic origins of magnetism.

The materials forming ferromagnetism can be divided into permanent (hard) magnetic materials and soft magnetic materials. Especially in hard magnetic materials there is no unique functionality between M (or J) and H, but the phenomenon of **hysteresis** takes effect. Fig. A1 shows the J(H) dependence (equivalent to the M(H) dependence, see definition of J above.) for soft and hard magnetic materials schematically. The doted curve is taken when the material comes from the demagnetized state, whereas the material uses the outer hysteresis after being fully magnetized in one direction.

Soft magnetic materials show also a hysteresis, but having a width being much narrower (up to a factor 1000 or more) compared to permanent magnets. So in fig. l the J(H) relation seems to be one unique line.