

## A. Basic Parameters of Permanent Magnetism

Magnetic materials can be described as an assembly of magnetic moments, which are the source of magnetic fields. The magnetic moment is a vector quantity. Summing up all magnetic moments and dividing by the volume results in the so called magnetization vector. The magnetic moment can be considered analogous to the electric charge in electricity. Magnetization would then be equivalent to the electric charge density. So we have as basic carriers of magnetism:

$$\begin{aligned} \vec{m} & \quad \text{Magnetic Moment [Am}^2\text{]} \\ \vec{M} = \frac{d\vec{m}}{dV} & \quad \text{Magnetization [A/m]} \end{aligned} \quad (\text{A.1})$$

Instead of the magnetization vector literature and technicians often use the so called magnetic polarization  $\vec{J}$ , which differs from  $\vec{M}$  only by the permeability constant  $\mu_0$ :

$$\vec{J} = \mu_0 \cdot \vec{M} \quad [\text{T}] \quad \text{with} \quad \mu_0 = 1.257 \cdot 10^{-6} \text{ Vs/Am} \quad (\text{A.2})$$

The resulting fields in magnetism can be divided into:

$$\begin{aligned} \vec{B} & \quad \text{magnetic Flux Density [T]} \\ \text{and} \\ \vec{H} & \quad \text{magnetic Field Intensity [A/m]} \end{aligned}$$

The description of fields and resulting phenomena like forces, torques, energies etc. is done using the so called **Maxwell equations**:

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (\text{Flux conservation}) \quad (\text{A.3})$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \quad (\text{Ampere's Law}) \quad (\text{A.4})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{Faraday's Law}) \quad (\text{A.5})$$

$$\vec{\nabla} \cdot \vec{D} = \rho \quad (\text{Coulomb Law}) \quad (\text{A.6})$$

$\vec{j}$  is the current density [A/m<sup>2</sup>] and  $\vec{E}$  denotes the electric field strength [V/m]. The electric flux density  $\vec{D}$  has the unit [As/m<sup>2</sup>].  $\rho$  describes the electric charge density [As/m<sup>3</sup>].

How enter magnetic moments or the magnetization vector these Maxwell equations? This happens by the so called **constitutive relation**, which provides the relation between fields **B**, **H** and magnetization **M**:

$$\vec{B} = \mu_0 \cdot \vec{H} + \mu_0 \cdot \vec{M} \quad (\text{A.7})$$

By use of **J** instead of **M** this becomes:

$$\vec{B} = \mu_0 \cdot \vec{H} + \vec{J} \quad (\text{A.8})$$

The behavior of **M**, especially its dependence of **H** provides the division of magnetism into its basic phenomena. The **H** dependence is described by the so called susceptibility  $\chi$  which is a unitless parameter:

$$\vec{M} = \chi(\text{H}) \cdot \vec{H} \quad (\text{A.9})$$

Relative permeability can be defined by

$$\mu = \chi + 1 \quad (\text{A.10})$$

By this one gets from eq. (A.2), (A.8) and (A.9):

$$\vec{B} = \mu_0 \cdot \mu \cdot \vec{H} \quad (\text{A.11})$$

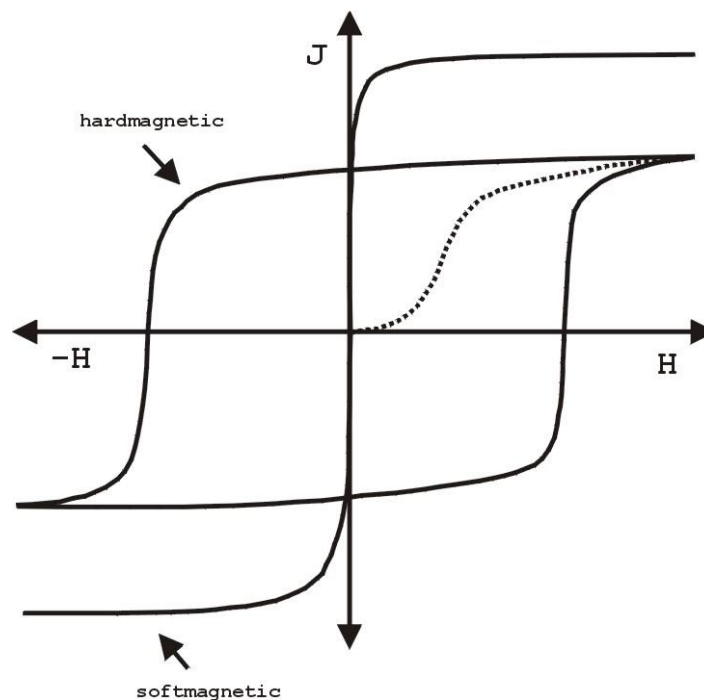


Fig.A1: Hard magnetic and soft magnetic materials and their  $J(H)$ -dependence. Doted curve: Virgin magnetization curve.

We make a rough division of magnetism into base classes now by:

$$\text{Diamagnetism: } \chi < 0, |\chi| \ll 1 \quad (\text{A.12a})$$

$$\text{Paramagnetism: } \chi > 0, |\chi| \ll 1 \quad (\text{A.12b})$$

$$\text{Ferromagnetism: } \chi > 0, |\chi| \geq 1 \quad (\text{A.12c})$$

Technically dia- and paramagnetism can be neglected in most cases. Their sources will be described in the chapter about atomic origins of magnetism.

The materials showing ferromagnetism can be divided into permanent (hard) magnetic materials and soft magnetic materials. Especially in hard magnetic materials there is no unique functionality between  $\mathbf{M}$  (or  $\mathbf{J}$ ) and  $\mathbf{H}$ , but the phenomenon of **hysteresis** takes effect. Fig. A1 shows schematically the  $J(H)$  dependence (equivalent to the  $M(H)$  dependence, see definition of  $\mathbf{J}$  above.) for soft and hard magnetic materials. The dotted curve occurs when the material comes from the demagnetized state, whereas the material travels along the outer hysteresis after being magnetized in one direction into saturation.

Soft magnetic materials show also a hysteresis, but those have a width being much narrower (up to a factor 1000 or more) compared to permanent magnets. So in fig. 1 the  $J(H)$  relation seems to be one unique line.